# THERMOHYDRAULIC STABILITY OF HELIUM FLOW AT SUPERCRITICAL PRESSURE UNDER CONDITIONS OF FORCED AND MIXED CONVECTION IN A VERTICAL HEATED CHANNEL

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The excitation of thermally induced oscillations in low-temperature helium flow in channels with different orientation are studied and the boundaries of stability are constructed.

It is well known that when one- and two-phase flows are heated in heat exchangers spontaneous pulsational flow regimes are possible for a certain combination of working parameters, including in low-temperature helium flows at supercritical pressure [1-3]. Oscillations of the flow rate, temperature, and pressure of the heat-transfer agent have a direct effect on the operating charactistics of power-generating equipment and, in many cases, are completely inadmissible.

To achieve high reliability of cryogenic equipment, cooled with a circulating helium flow at a temperature of 4.2-10 K, the conditions of excitation of oscillations must be studied and measures must be developed to increase the stability of cooling.

The experiments of [2] and later the calculations of [4] for the case of purely forced convection of helium demonstrated that the boundary of stability can be represented in the form of dependence $\psi = f(R)$, where $\psi = (\Delta p_1 + \Delta p_{1m})/(\Delta p_{2m} + \Delta p_2)$; $R = (\rho_{in} - \rho_{out})/\rho_{in}$.

The parameter $\psi$ is defined as the ratio of the pressure losses on separate sections of the channel separated by the cross section where the temperature of the liquid equals the pseudocritical temperature ($T_{liq} = T_m$), and is the analog of the well-known criterion of P. A. Petrov for evaluating the stability of two-phase flows [5]. The other parameter R characterizes the degree of volume expansion of the working body when the body is heated. The calculations and experiments were performed for a pipe with a constant length L = 185 m [the reduced length $\bar{L} = \xi(L/D) \simeq 1000$] and the resistance coefficients of the throttles ($K = G^2/\rho u^2$) at the inlet and outlet of the channel $K_{in}$ = 120-2700 and $K_{out}$ = 1000-23,700.

It is not obvious that $\psi$ and R can be used to evaluate the stability of the flow of liquid in a mass force field. This follows from the fact that for flow in vertical pipes the magnitude of the leveling heads can be comparable and even greater than the pressure drops owing to frictional hydraulic losses. In this case the quantity $\psi$ will depend more on the ratios of the leveling heads along the channel than on the hydraulic losses.

The relative volume expansion of the gas in the channel in its turn depends not only on the heat load ($q_w/\rho u$) but also on the magnitude of the pressure drop, which is especially important in long channels [$\bar{L} = \xi(L/D)$]. The effect of the length and of the absolute values of the resistance coefficients of the throttles $K_{in}$ and $K_{out}$ at the channel inlet and outlet on the stability under conditions of forced convection of helium was studied in [6]. It was shown that for $K_{in} = K_{out} = 0$ the stability is virtually independent of $\bar{L}$, while the boundary of stability approaches the limiting value R = 10.

The purpose of this work is to make a comparative analysis of the stability of helium flow at supercritical pressure under conditions of purely forced convection with rising and sinking flow for a wide range of values of the numbers Re = $10^3$-$10^5$ and $Gr_A/Re^2 = 3.0\cdot10^{-7}$-$1.1\cdot10^{-1}$, i.e., including the region where heat transfer is observed to depend appreciably on thermogravitation [3].

The dynamic processes were studied with the help of the classical method of the theory of linear automatic-control systems [7]. The mathematical model of the processes of interest

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consists of the following equations:

$$\frac{D\rho}{Dt}=0; \tag{1}$$

$$\rho\frac{Dh}{Dt}=q_w\frac{\Pi}{F}+\frac{dp}{dt}; \tag{2}$$

$$\rho\frac{Du}{Dt}=-\frac{dp}{dx}-\xi\frac{\rho u^2\Pi}{2F}+mg\rho, \tag{3}$$

where m = 1 for the rising flow and m = −1 for the sinking flow;

$$\rho=f(h,\ p). \tag{4}$$

The coefficient of friction was determined from the relation [8]

$$\xi=(1{,}82\lg \mathrm{Re}-1{,}64)^{-2}. \tag{5}$$

The boundary conditions at the ends of the channel are taken in the form

$$x=0:\Delta p_1=p_0-p_{\text{in}}=K_{\text{in}}\rho_0u_0^2,\ \ x=L:\Delta p_2=p_N-p_{\text{out}}=K_{\text{out}}\rho_{\text{out}}u_{\text{out}}^2\ . \tag{6}$$

It is assumed that the pressure at the pipe inlet in front of the throttle and at the pipe outlet as well as the temperature at the pipe inlet are maintained constant.

We shall study the stability of a dynamic system with distributed parameters "in the small," i.e., the stability of the system of equations (1)-(5) in the linear approximation. In performing the calculations the heat flux from the pipe wall to the helium was assumed to be constant; this is a good approximation for low frequencies of oscillations of the temperature of the wall with thickness b:

$$\omega\ll\frac{\lambda_w}{\rho_w c_w}\frac{1}{b^2}. \tag{7}$$

The traditional self-excited oscillatory system includes an energy source, a valve, an oscillatory system, and feedback on the valve; the energy source and the valve are both placed in the input valve, while the kinetic energy of the flow is the energy source. The feedback (in this case, on the pressure perturbation after the input valve $\delta p_1$) connects the oscillatory system (the equations of conservation and state and the boundary condition at the output from the pipe) with the flow rate of the working body at the input and changes the kinetic energy of the flow, i.e., it controls the energy input to the oscillatory system. When the pressure after the input valve increases with time, the flow rate decreases and therefore the kinetic energy of the flow decreases as compared with the stationary flow. The decrease in the kinetic energy later leads to a decrease in the pressure after the input valve. It is obvious that this relation between the perturbations of the pressure and the flow rate for some ratio of the phases of the oscillations, determined by the feedback, can lead to excitation of the system.

We set $u=\bar{u}+\delta u$, $p=\bar{p}+\delta p$, $h=\bar{h}+\delta h$, where $\bar{u}$, $\bar{p}$, and $\bar{h}$ are the values of the flow parameters distributed along the pipe under stationary conditions.

After linearization the equations (1)-(4) assume the following form:

$$\frac{\partial\delta\rho}{\partial t}+\bar{\rho}\frac{\partial\delta u}{\partial x}+\delta\rho\frac{\partial\bar{u}}{\partial x}+\bar{u}\frac{\partial\delta\rho}{\partial x}+\delta u\frac{\partial\bar{\rho}}{\partial x}=0, \tag{8}$$

$$\frac{\partial\delta h}{\partial t}+\bar{u}\frac{\partial\delta h}{\partial x}+\delta u\frac{\partial\bar{h}}{\partial x}+\frac{\bar{u}}{\bar{\rho}}\delta\rho\frac{\partial\bar{h}}{\partial x}-\frac{\bar{u}}{\bar{\rho}}\frac{\partial\delta p}{\partial x}-\frac{\delta u}{\bar{\rho}}\frac{\partial\bar{p}}{\partial x}-\frac{1}{\bar{\rho}}\frac{\partial\delta p}{\partial t}=0, \tag{9}$$

$$\bar{\rho}\frac{\partial\delta u}{\partial t}+\bar{\rho}\delta u\frac{\partial\bar{u}}{\partial x}+\delta\rho\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{\rho}\bar{u}\frac{\partial\delta u}{\partial x}+\frac{\partial\delta p}{\partial x}+\frac{1}{2}\frac{\xi\Pi}{F}(2\bar{\rho}\bar{u}\,\delta u+\bar{u}^2\delta\rho)-mg\delta\rho=0, \tag{10}$$

$$\delta\rho=\left(\frac{\delta\rho}{\delta h}\right)_p\delta h+\left(\frac{\partial\rho}{\partial p}\right)_h\delta p. \tag{11}$$

Because the properties of helium in the near-critical region are strongly nonlinear functions of the temperature and the pressure, Eqs. (8)-(11) are written in the difference form with respect to x. The number of partitions is determined from the transfer function of the open contour, determined with adequate accuracy, within ±1% and by the degree of variation of the parameters $\bar{p}$, $\bar{h}$, and $\bar{u}$ along the channel for a stationary flow. Then

$$\delta f = \frac{\delta f_j + \delta f_{j+1}}{2}, \quad \frac{d\delta f}{dx} = \frac{\delta f_{j+1} - \delta f_j}{\Delta x},$$

where f = h, u, and p; j = 1, ..., N.

We have the following boundary conditions

$$\delta(\Delta p_1) = -\delta p_1 = 2K_{in}\rho_0 u_0 \delta u_1, \tag{12}$$

$$\delta(\Delta p_2) = \delta p_N = \frac{-K_{out}(\bar{\rho}\bar{u})^2 \left(\frac{\partial v}{\partial h}\right)_p \delta h_N + 2K_{out}\bar{\rho}\bar{u}\delta u_N}{1 + K_{out}\bar{\rho}\bar{u}\left(\frac{\partial v}{\partial p}\right)_h}. \tag{13}$$

In the derivation of Eq. (12) the fact that in the absence of heat inflows at the valve inlet and for not very strong narrowing of the passage in the valve as compared with the cross section of the pipe $\delta u_i \simeq \delta u_0$ was taken into account. In the case of large over compression of the flow a corresponding constant factor, greater than unity, must be introduced into Eq. (12).

The stability of a dynamic system with distributed parameters and with delay is analyzed using Nyquist's frequency criterion based on the argument principle [7]. This method was employed, in particular, to analyze the thermohydraulic stability of helium flow at supercritical pressure under conditions of purely forced convection [4]. It was shown that the computational and experimental results are in satisfactory agreement.

After the linearized system of equations (8)-(10) with the boundary conditions (11)-(12) is Laplace transformed and the equations are represented in a difference form, and setting the starting perturbation at the input into the channel $\delta u_1 = 1$, we obtain a system of 3N linear algebraic equations, whose solution for the functions $\delta u_j$, $\delta p_j$, and $\delta h_j$ is sought by the matrix sweep method [9].

The transfer function of feedback $H(s) = \delta p_1/\delta u_1$ – the ratio of the perturbations of the pressure and the velocity of the liquid at the channel inlet – can be determined from the solution of the system of equations. The transfer function for direct coupling is determined from Eq. (12) for the inlet throttle

$$J(s) = 0{,}5\,(K_{in}\bar{\rho}_0\bar{u}_0)^{-1}.$$

The transfer function of an open loop equals the product J(s)H(s). The stability of a dynamic closed system is determined by the position of the hodograph of the amplitude–phase frequency charcteristic of an open loop with the parameter $s = i\omega$ relative to the singular point (–1, i0).

It is obvious that for relatively short pipes [$K_{in}$, $K_{out} \gg \xi(L/D)$], neglecting the leveling head,

$$\Psi = \Psi_1 \simeq \frac{K_{in}}{K_{out}} \frac{\rho_{out}}{\rho_{in}}. \tag{14}$$

In this case the boundary of stability can be constructed in the system of coordinates $\bar{K} = K_{in}/K_{out} = f(R)$.

Figure 1 shows the boundary of stability for helium flow under conditions of purely forced convection in a relatively short channel ($\bar{L} < 50$) in terms of the function $\bar{K} = f(R, K_{out})$. To the right of the boundary pulsational flow regimes can appear. For a pipe open at the outlet (no throttle), under conditions of turbulent flow $K_{out} \simeq 2$. In the region $\bar{K} <$

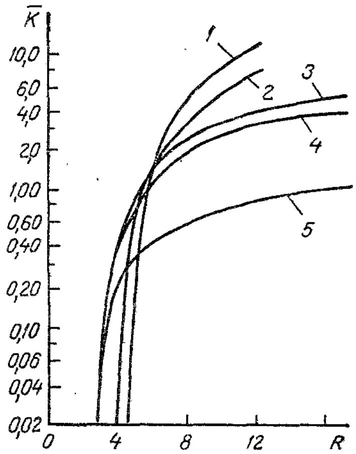


Fig. 1

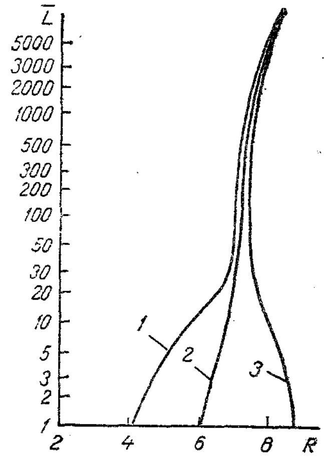


Fig. 2

Fig. 1. Map of the stability under conditions of forced convection, $L = 0.6$ m; $D = 1.8 \cdot 10^{-3}$ m; $Re_{in} = 10^4$; $T_{in} = 4.2$ K; $p = 0.25$ MPa; $K_{out} = 1$ (1); 2 (2); 100 (3); 500 (4); $10^4$ (5).

Fig. 2. Map of the stability under conditions of forced convection, $D = (1.8\text{-}4) \cdot 10^{-3}$ m; $Re_{in} = 10^4$; $T_{in} = 4.2$ K; $p = 0.25$ MPa; 1) $K_{out}(K_{in}) = 2$ (1); 2) 2 (4); 3) 2 (8).

1.5, which includes the case of a channel open at both ends and with sharp rims without throttles ($\bar{K} = 0.5$), as $K_{out}$ increases the boundary of stability shifts toward smaller values of R and reaches the limiting value $R = 2.8$. In the region $\bar{K} > 1.5$ the boundary of stability shifts toward larger values of R, and the larger $K_{out}$ is the lower the relative throttling for which a stable state can be obtained.

It should be noted that enlarging the region of stability by increasing the resistance of the throttle at the outlet with constant $\bar{K}$ involves an increase in the overall hydraulic losses, which vary under otherwise equal conditions at the inlet in the ratio

$$\Delta p \sim K_{out} (\bar{K} + R + 1). \quad (15)$$

For this reason, for example, the total hydraulic losses accompanying expansion of the flow $\bar{R} = 12$ at the boundary of stability will be two orders of magnitude larger for $K_{out} = 500$ ($\bar{K} = 4$) than for $K_{out} = 2$ ($\bar{K} = 12$) (Fig. 1).

The effect of the channel length on the stability is not single-valued (Fig. 2). For small values of $K_{out}$ the stability in the region $\bar{L} > 30$ is virtually independent of the length and the ratio $\bar{K}$. For $\bar{L} < 30$ and $\bar{K} > 2.5$ the stability increases as the length increases, while for $\bar{K} < 2.5$, conversely, it decreases. The dependence of the stability on the channel length is shown in Fig. 3 for a wide range of values of $K_{out}$. In the region of small $\bar{K}$ increasing the channel length leads to significant stabilization of the flow, and in addition the dependence on $K_{out}$ is insignificant. For $\bar{K} > 2$, on the other hand, the hydraulic resistance at the output from the channel has a stronger effect on the thermohydraulic stability.

The representation of the computational results in terms of the coordinates Ψ–R qualitatively describes the same character of the dependence of the stability on the flow and channel parameters; only the boundary of stability in the direction of the ordinate axis is deformed. The correlation coefficient between $\bar{K}$ and the boundary of stability, determined from the hodograph of the function H(s)J(s), is not worse than for the parameter Ψ, including experimental data [2]. The effect of the position of the point with the pseudocritical temperature $T_{liq} = T_m$ relative to the channel inlet and outlet on the boundary of stability was insignificant (for the same degree of expansion of helium). It should be noted that the calculations were performed for temperatures at the inlet $T_{in} > 4$ K. The cross section of the channel with $T_{liq} = T_m$ for regimes on the boundary of stability was located close to the channel input, while the hydraulic resistance was determined primarily by the pressure losses in the pipe after the cross section with $T_{liq} = T_m$ and on the throttling devices.

In the experimental study of thermally induced oscillations in a helium flow at supercritical pressure in [2] the resistance coefficient $K_{out}$ varied over a wide range $K_{out} = (1$-

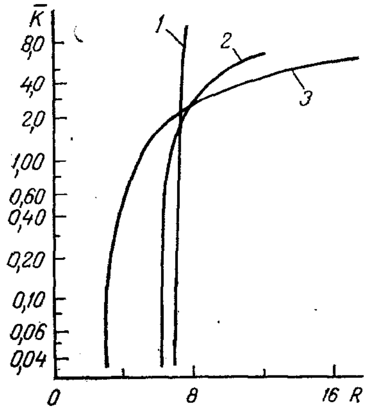


Fig. 3

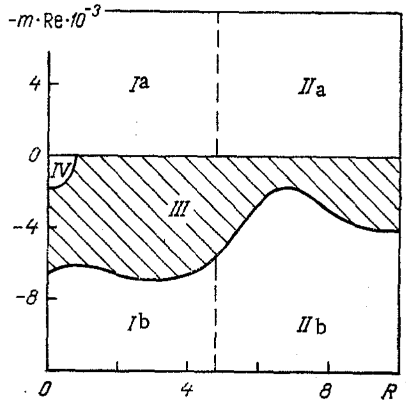


Fig. 4

Fig. 3. Map of the stability under conditions of forced convection $p = 0.25$ MPa; $R_{in} = 10^4$; $T_{in} = 4.2$ K; 1, 2) $L = 185$ m, $D = 4 \cdot 10^{-3}$ m, $K_{out} = 2$ and 100 respectively; 3) $L = 0.6$ m, $D = 1.8 \cdot 10^{-3}$ m, $K_{out} = 100$.

Fig. 4. Map of the stability under conditions of mixed and forced convection in a vertical pipe, $p = 0.25$ MPa; $T_{in} = 4.2$ K; $L = 0.6$ m; $D = 1.8 \cdot 10^{-3}$ m; $K_{in} = 1$; $K_{out} = 2$: I, IV) regions of stable states; II) regions of pulsational states; III) region of aperiodic instability; a) rising flow, b) sinking flow; the broken line is the boundary of stability (of pulsational regimes) with forced and mixed convection.

23.7)$\cdot 10^3$, and $\bar{K} = 0.2 \cdot 269$. Based on the dependence of the boundary of stability on the absolute value of $K_{out}$, presented in [2], the boundary of stability must be regarded as approximate, especially in the region of large volume expansions ($R > 6$).

For helium flow in vertical pipes for Reynolds numbers $Re > 8000$ the boundary of stability does not depend on the direction of flow and is the same as for purely forced convection (Fig. 4), i.e., it is determined by the ratio of the hydraulic resistances at the ends of the pipe and the variation of the helium density along the channel and does not depend on the number Gr. Analogous results regarding the independence of the boundary of stability from thermogravitation under conditions of rising helium flow in a vertical pipe were noted in [3]. The boundary of stability in the experiments $R \simeq 1$, which is less than in the calculations ($R_{min} = 2.8$). This discrepancy can, on the one hand, be associated with the characteristics of the heat-exchange flow through part of the experimental apparatus and the existence of additional feedbacks through the heat exchangers; on the other hand, the region at the wall was not taken into account in the calculations based on the one-dimensional model of the flow. For large departures from isothermal conditions the region at the wall can have a destabilizing effect; this is manifested as a decrease in the stability as the length of the heated section decreases for equal degrees of volume expansion of the gas [10].

For low Reynolds numbers Re the stability depends on the direction of the flow. For sinking flow, conditions for aperiodic instability (region III, Fig. 4) arise; this corresponds to the point on the hodograph of the amplitude–phase frequency characteristic of an open loop with $\omega = 0$. As is well known, aperiodic instability arises with a negative dependence of the change in the hydraulic resistance of the heated section of the pipe on the change in the flow rate ($dp_{in}/dG < 0$). The change in the sign of $dp/dG$ as the flow rate increases under the conditions of sinking motion characterizes the multi-valued nature of the hydraulic characteristic in the presence of a leveling head, whose vector is directed opposite to the vector of the pressure drop for overcoming the friction forces.

The appearance of aperiodic instability in loops with a heated sinking branch was noted in many works on the flow of water in a two-phase state and under supercritical pressure [11]. It was also noted that reverse flow of water is the most dangerous type of instability. A unique flow rate was not obtained, if the flow rate dropped below the corresponding minimum of the hydraulic characteristic $\Delta p = f(G)$, while the pressure drop in the loop was close to the minimum values.

## NOTATION

c, heat capacity; D, diameter; F, area of the transverse cross section; G, flow rate; $Gr_A$, Grashof number; g, acceleration of gravity; h, enthalpy; p, pressure; Π, perimeter; Re, Reynolds number; s, complex Laplace transform variable; T, temperature; t, time; x, distance along the channel; λ, thermal conductivity; ω, circular frequency of the pulsations; and ρ, specific density. The indices denote the following: in, at the inlet to the channel; out, at the outlet from the channel; liq, the flow; 0, in front of the throttle at the channel input; m, the pseudocritical temperature; N, in front of the throttle at the channel output; w and c, the pipe wall.

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# UNIVERSAL PROFILES AND LAW OF TURBULENT NEAR-WALL HEAT AND MASS TRANSFER


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UDC 532.526



Universal distributions that do not contain empirical constants are obtained in the turbulent core of the mean longitudinal velocity, temperature and concentration of a substance for arbitrary Prandtl and Schmidt molecular numbers.


The development of modern engineering in the domain that is characterized by the presence of internal or external heat and mass transfer on streamlined surfaces is greatly retarded because there is no single description of this phenomenon for different values of the Pradtl and Schmidt numbers based on universal distributions of the velocity, temperature, and concentration of a substance that do not contain empirical constants, and a law of turbulent heat and mass transfer. Precisely the absence of experimental coefficients in such generalized relationships permits their effective application in computations of complex flow conditions characteristic for energy-saving aggregates, consequently, setting up universal dependences is of great scientific and practical interest. An attempt is made in this paper to obtain such generalized relationships and the possibility is shown of their utilization to describe specific flows.


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